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A Qualitative Spatio-Temporal Framework Based On Point Algebra

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What is Qualitative Reasoning?

- Qualitative Reasoning is based on qualitative abstractions of aspects of the common-sense background knowledge, such as *space* and *time*, on which our human perspective on the physical reality is based.

Applications of Qualitative Reasoning

- Qualitative reasoning is an important subproblem in many *applications*, such as:
  - Problem Solving
  - Dynamic GIS
  - Cognitive robotics
  - Planning
  - SpatioTemporal representation and reasoning

Reasons for Qualitative Reasoning

- Main reasons why non-precise, qualitative information may be useful:
    1. Only partial information may be available
    2. Constraints are often most naturally stated in qualitative terms
    3. Abstraction from numeric quantities boosts research and applications

Reasoning about time

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- The Point Algebra (PA) is one of the dominant Artificial Intelligence approaches for representing and reasoning about qualitative temporal relations
- PA encodes temporal relations between two points in the timeline using the set of base relations $\{<, =, >\}$
- PA forms the basis of several richer temporal languages, such as Interval Algebra (IA)

The IA Constraint Language

- Interval Algebra (IA) [2] encodes the possible binary relations between time intervals in a timeline

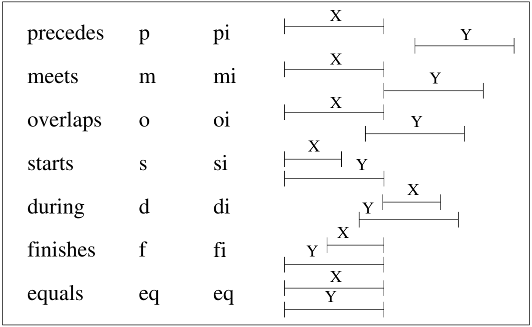

Figure: The thirteen base relations of IA

Reasoning about space

- A fragment of the Region Connection Calculus, namely, RCC-8, Cardinal Direction Algebra (CDA), and Rectangle Algebra (RA), are among the dominant Artificial Intelligence approaches for representing and reasoning about qualitative spatial relations.
- RCC-8 encodes topological relations between two regions that are non-empty regular subsets of some topological space

The RCC-8 Constraint Language

- RCC-8 is a fragment of the Region Connection Calculus (RCC) [1]
- RCC-8 encodes binary topological relations between regions that are non-empty regular subsets of some topological space

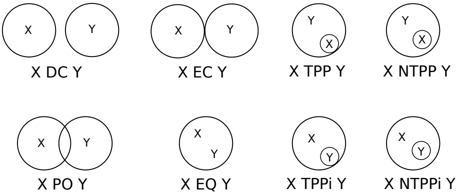

Figure: Two dimesional examples for the eight base relations of RCC-8

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The RSAT Reasoning Problem

- RSAT is the reasoning problem of deciding whether a temporal or spatial network is satisfiable by a spatial or temporal configuration Θ respectively
- RSAT is NP-Complete for the considered calculi, expect PA for which it is polynomial [3, 4]
- However, tractable signatures of the calculi exist for which the consistency problem can be decided in polynomial time with a *path consistency* algorithm [3, 4]

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Path Consistency

- Approximates consistency and realizes *forward checking* in a backtracking algorithm
- Checks the consistency of triples of relations and eliminates relations that are impossible though iteravely performing the operation

  $$R_{ij} \leftarrow R_{ij} \cap R_{ik} \diamond R_{kj}$$

  until a fixed point $\overline{R}$ is reached
- If $R_{ij} = \emptyset$ for a pair (i, j) then R is inconsistent, otherwise $\overline{R}$ is *path consistent*.
- Computing $\overline{R}$ is upper bounded by $O(n^3)$ time

Reasoning about spacetime

- Formalisms based on propositional temporal logic (PTL)+ RCC-8, e.g., $\Box EC(X, Y)$
- Multimodal logic approaches on velocity, movement, and other spatiotemporal aspects
- A single constraint based formalism where IA is combined with RCC-8 by Gerevini and Nebel, called STCC

The STCC formalism

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- A spatial network is associated to every variable of a network of Interval Algebra (IA)

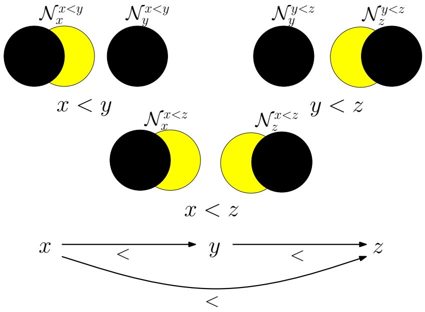
- The IA relation implied between x and y is $\{p, m, mi, pi\}$, and if taken together with the basic temporal relations leads to the full algebra when closing the relations under intersection, converse, and composition
- Satisfiability problem of STCC is $\mathcal{NP}$-hard even if CSPs contain basic relations and the two universal relations

Motivation

- Provide a richer framework
- Allow for more interesting tractability cases
- Be able to define laws about qualitative change, such as movement

Proposed Framework - QSTCN

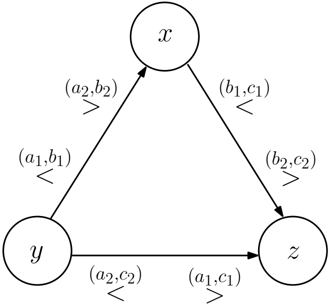
- A pair of spatial networks is associated to every base relation of a network of Point Algebra (PA)



- Upon instatiation of a base relation, the spatial networks associate themselves to the corresponding variables

Extracting a spatial network

- When considering atomic spatial QCNs, we can view them as constant unique values of a CSP



- Variables: $x : \{b_1, b_2\}$, $y : \{a_1, a_2\}$, $z : \{c_1, c_2\}$
  Constraints: $R_{yx} : \{(a_1, b_1), (a_2, b_2)\}$,
  $R_{yz} : \{(a_2, c_2), (a_1, c_1)\}$, $R_{xz} : \{(b_1, c_1), (b_2, c_2)\}$

Constraint Propagation

- We need to propagate constraints between the temporal and the spatial aspect of a QSTCN
- Compositions of relations between the spatial and temporal aspect must be intersected per pair of base relations

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ST-Path Consistency Algoritm

Algorithm 1: stPC($\mathcal{N}$, $\mathcal{N}_\mathsf{S}$)

in : A QSTCN $\mathcal{N} = (V_\mathsf{T}, V_\mathsf{S}, C, \alpha)$, and CSP $\mathcal{N}_\mathsf{S} = \langle X, D, R \rangle$.

output : False if network $\mathcal{N}$ results in a trivial inconsistency (contains the empty relation), True if the modified network $\mathcal{N}$ is path consistent.

begin
- $Q \leftarrow \{(i,j) \mid (i,j) \in V_\mathsf{T} \times V_\mathsf{T}\}$;
- **while** $Q \neq \emptyset$ **do**
  - $(i,j) \leftarrow Q.pop()$;
  - **foreach** $k \leftarrow 1$ **to** V_T, $(i \neq k \neq j)$ **do**
    - $t \leftarrow C(i,k) \cap ((C(i,j) \diamond C(j,k)) \cap \alpha^{-1}(r(i_s, j_s) \circ r(j_s, k_s)))$;
    - **if** $t \neq C(i,k)$ **then**
      - **if** $t = \emptyset$ **then return** False;
      - $C(i,k) \leftarrow t$; $C(k,i) \leftarrow t^{-1}$;
      - $Q \leftarrow Q \cup \{(i,k)\}$;
    - $t \leftarrow C(k,j) \cap ((C(k,i) \diamond C(i,j)) \cap \alpha^{-1}(r(k_s, i_s) \circ r(i_s, j_s)))$;
    - **if** $t \neq C(k,j)$ **then**
      - **if** $t = \emptyset$ **then return** False;
      - $C(k,j) \leftarrow t$; $C(j,k) \leftarrow t^{-1}$;
      - $Q \leftarrow Q \cup \{(k,j)\}$;
- **return** True;

Example

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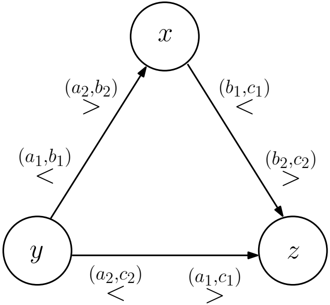
- The following QSTCN is inconsistent



- $(Cxy \diamond Cyz = \{\overset{(a_1,c_1)}{<}, \overset{(a_2,c_2)}{>}\}) \cap$ $(\alpha^{-1}(r_{xy} \circ r_{yz}) = \{\overset{(a_1,c_1)}{>}, \overset{(a_2,c_2)}{<}\}) = \emptyset$

Complexity Results

Proposition

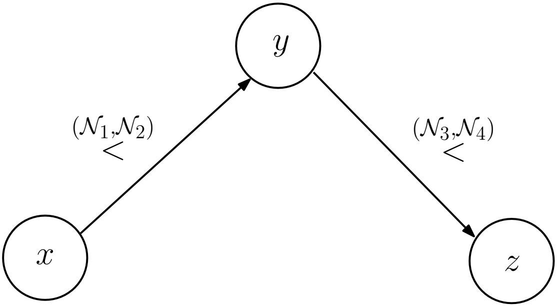
Given a path consistent atomic QSTCN $\mathcal{N}$, $\mathcal{N}$ *is tractable*

Proposition

Given a path consistent QSTCN $\mathcal{N}$ *where the spatial* QCNs *are atomic and the underlying* QCN *of* PA *has relations from the convex class of relations* $\{\emptyset, <, =, >, \leq, \geq, = \vee \neq\}$, $\mathcal{N}$ *is tractable*

General Case

- We can consider general, non atomic, spatial QCNs



- We have to consider the intersection of $\mathcal{N}_2$ and $\mathcal{N}_3$ (e.g., $X\{EC \vee DC\}Y \cap X\{DC \vee TPP\}Y = X\{DC\}Y$)

Complexity Result for General Case

Proposition

Given a QSTCN $\mathcal{N}$ *where the underlying* QCN *of* PA *atomic and the associated spatial* QCNs *are non atomic, solving* $\mathcal{N}$ *has the same complexity with solving the associated spatial* QCNs

Main Points

- Definition of a new qualitative constraint-based spatiotemporal framework using Point Algebra (PA)
- Study of the computational properties of derived formalisms

Main Directions

- Further explore cases of tractability, especially for QSTCNs that comprise non atomic spatial QCNs.
- Create and experiment with a benchmark of constraint-based qualitative spatiotemporal instances

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The End

Any Questions?